# $SnowDepth^{\rm TM}$

Theory Manual

# Contents

1	Introduction			
2	Theory			
	2.1	Rair	n or Snow	
	2.2	Sno	w Accumulation	
	2.3	Ene	rgy Exchange at Snow-Air Interface	
	2.3.	1	Rain on Snow Melt	
	2.3.2	2	Non-Rain Melt	
	2.3.	3	Energy Exchange when No Surface Melt	
	2.4	Inter	rnal Changes within Snow Pack9	
	2.4.	1	Snow Cover Ripeness	
	2.4.2	2	Snow Depth Calculation	
3	Sno	wDep	pth Calibration15	
	3.1 Summary of Parameters			
	3.1.	1	Major Snow Parameters:	
	3.1.2	2	Minor Snow Parameters17	
4	Vali	idatio	on19	
	4.1	Disc	cussion	
5	Refe	erenc	es27	

# **1** Introduction

The SnowDepth<sup>TM</sup> software, which is based on the SNOW-17 theory, as described by Eric Anderson [1] is a snow accumulation and ablation model used to estimate the depth of snow on natural surfaces. It is a conceptual model in which each of the significant physical processes affecting snow accumulation and snowmelt are mathematically represented. The model uses air temperature as the sole index to energy exchange across the snow-air interface and was originally developed to run in conjunction with a rainfall-runoff model.

A novel approach implemented in this software is the use of constrained non-linear least-squares optimization to calibrate model parameters for a particular location. This greatly enhances its accuracy in comparison to a standard implementation of SNOW-17.

Some validation is provided in the final section of this documentation.

# 2 Theory

SNOW-17 is an index model using air temperature as the sole index to determine the energy exchange across the snow-air interface. In addition to temperature, the only other input variable needed to run the model is precipitation. It was originally designed for use in river forecasting but has successfully been applied at point locations to simulate the accumulation and melting of the snow cover.

For river forecasting, large river basins are divided into headwaters and local areas generally based on where river observations are available. In flat terrain, SNOW-17 is typically applied to a headwater drainage or local area, though in some cases large drainages may be divided into several sub-areas. In mountainous regions, due to the significant variation in the amount of snow and the timing of melt with elevation, watersheds are typically divided into 2 or 3 elevation zones when using SNOW-17. Since the model was not designed to calculate how melt rates might vary with various physiographic factors SNOW-17 is not generally used for applications such as predicting the effect of land use changes.

In order to get the best results from SNOW-17 for river forecasting applications three things must occur:

- 1. The model must be properly calibrated,
- 2. The input data (precipitation and temperature) used operationally must be unbiased compared to that used for calibration, and
- 3. Well devised, ideally objective, updating schemes must be used to remove bias and to minimize random errors to the maximum extent possible.

The values of the model parameters, as determined through calibration, represent normal conditions over a river basin (i.e. the typical spatial variation of precipitation and temperature, the prevailing storm directions and wind conditions that affect the 3 measurement and distribution of snow, the typical climatological conditions during periods of melt, etc.). While many of the errors during calibration are random, there are certain biases that can't be overcome when using a temperature index snow model (e.g. very high melt rates, often associated with major runoff events, are generally the result of abnormal meteorological conditions, such as high winds and dew-points, resulting in a tendency to under estimate snowmelt during such events). Figure 1 shows the model diagram for the SNOW-17 algorithm that is implemented in SnowDepth. The following sections explain each stage in detail. Areal Extent of the Snow Cover and Transmission of Excess Liquid are not implemented in SnowDepth as the outflow is not calculated.



Figure 1 – SNOW-17 Model Diagram

## 2.1 Rain or Snow

PXTEMP is an input parameter which represents the threshold air temperature above which all precipitation is classified as rain.

## 2.2 Snow Accumulation

$$P_n = P \cdot f_s \cdot SFC \tag{1}$$

where: Pn = water equivalent of new snowfall (mm),

P = total precipitation input to the model (mm), and

 $f_s$  = fraction of precipitation in the form of snow.  $f_s$  is fixed at 1.

SFC accounts for gage catch deficiencies that occur during snowfall measurement caused be sublimation and redistribution of blowing snow. It is an average value used over the complete accumulation period. When melt periods are preceded by only a couple snow storms, errors are much greater as variations from event to event catch deficiency do not cancel out. The density of new snow is decided be Eq. 2; the value declines rapidly as temperature decreases between 0 to -12 °C followed by little further decrease for colder temperatures [2].

$$\rho_n = 0.001 \left[ 67.92 + 51.25 \exp\left(\frac{T_a}{2.59}\right) \right]$$
(2)

where:  $\rho_n$  = density of new snowfall (g/cm<sup>3</sup>)

 $T_a = \text{air temperature } (^{\circ}C)$ 

The depth of new snow fall is then calculated using Eq 3.

$$H_n = \frac{P_n}{\rho_n} \tag{3}$$

where:  $H_n = depth of new snowfall (mm)$ .

 $P_n$  = precipitation (mm)

 $\rho_n$  = density of new snowfall (g/cm<sup>3</sup>)

The cold content added to the snow pack is calculated using Eq. 4; i.e. the amount of heat that must be added to bring the temperature of the new snow to 0 degrees.

$$\Delta D_p = -\frac{T_n \cdot P_n}{L_f/c_i} \tag{4}$$

where:  $\Delta D_p$  = change in the heat deficit due to snowfall (mm),

 $T_n$  = temperature of the new snow (°C),

- $L_f = latent heat of fusion (80 cal \cdot gm-1)$
- $c_i = specific heat of ice (0.5 cal gm-1 \cdot ^{\circ}C-1)$

## 2.3 Energy Exchange at Snow-Air Interface

The SNOW-17 model calculates surface melt in different ways depending on whether rain is occurring or not. Melt during rain-on-snow periods is computed differently than melt during non-rain periods because:

- the magnitude of the various energy transfer components tend to be quite different between the 2 situations,
- the dominant energy transfer components during rain-on-snow periods are known, and

• the seasonal variation in melt rates is generally quite different between non-rain and rain periods.

The model also keeps track of the heat deficit within the snow cover that develops when the temperature drops below 0°C. SNOW-17 expresses energy exchange in terms of mm, where an mm of energy is the amount of heat required to melt or freeze 1 mm of ice or water, respectively, at  $0^{\circ}C$  – approximately 8 cal/cm<sup>2</sup>. This makes it easy to compare the heat deficit to the amount of melt or rain water required to overcome the deficit.

## 2.3.1 Rain on Snow Melt

An energy-balance model is used for rain on snow modelling and the following assumptions are made:

- 1. Incoming solar radiation is negligible because overcast conditions generally prevail.
- 2. Incoming longwave radiation is equal to black body radiation (emissivity of 1.0) at the temperature of the cloud layer which should be reasonably close to the air temperature
- 3. Relative humidity is quite high (90% is assumed), and thus the wet bulb temperature is essentially equal to air temperature
- 4. Snow surface temperature is equal to  $0^{\circ}$ C
- 5. Melt is independent of the time of the year. This assumption would only be invalid when there is a definite seasonal variation in the wind speed during rain events.

Eq. 5 is the energy budget equation used. **UADJ** is the parameter used to indicate the average wind function. This is only used when the precipitation is greater than 0.25mm per hour.

$$M_{r} = \sigma \cdot \Delta t_{p} \cdot [(T_{a} + 273)^{4} - 273^{4}] + 0.0125 \cdot P \cdot f_{r} \cdot T_{r} + 8.5 \cdot \textbf{UADJ} \cdot \left(\frac{\Delta t_{p}}{6}\right) \cdot [(0.9 \cdot e_{sat} - 6.11) + 0.00057 \cdot P_{a} \cdot T_{a}]$$
(5)

where: Mr = melt during rain-on-snow time intervals (mm),

 $\sigma = \text{Stefan-Boltzman constant} - 6.12 \cdot 10{\text{-10 mm}}^{\circ}\text{K/hr},$ 

 $\Delta t_p$  = time interval of precipitation data (hours),

 $T_a = air temperature (^{\circ}C),$ 

 $273. = 0^{\circ}C$  on the Kelvin scale,

fr = fraction of precipitation in the form of rain,

 $T_r$  = temperature of rain (°C) – (=T<sub>a</sub> or 0°C, whichever greater),

UADJ = average wind function (mm/mb/6 hr),

 $e_{sat} = saturated vapor pressure at T_a (mb)$ 

 $P_a = atmospheric pressure (mb)$ 

The saturated vapor pressure is calculated using Eq. 6 and the atmospheric pressure by Eq. 7.

$$e_{sat} = 2.7489 \cdot 10^8 \cdot \exp(-4278.63/(T_a + 242.792)) \tag{6}$$

$$P_a = 33.86 \cdot (29.9 - 0.335 \cdot H_e + 0.00022 \cdot H_e^{2.4}) \tag{7}$$

where:  $H_e = elevation$  (meters).

Further details on the derivation of Eq. 5, 6 and 7 are provided in Appendix of the original Anderson thesis [1].

#### 2.3.2 Non-Rain Melt

SNOW-17 uses a melt factor to estimate the amount of surface snowmelt for no rain and very light rainfalls, less than 0.25mm/hr. The melt factor is based on results from an energy balance model. In Figure 3, the curve represents the fitted curve SNOW-17 would implement. The points that deviate considerably from the average are during periods when air temperature is slightly above freezing or above freezing nighttime periods.



Figure 2 - Seasonal Melt Factor Variation for conterminous United States

The melt factor is thus calculated using Eq. 8 with parameters MFMAX and MFMIN determining extreme values of the melt factors. In Figure 3, the variation is shown for Alaska where minimal sunlight persists for most the winter. To account for the rapid increase and decline in melt from rapidly changing daylight hours, the factor  $A_{\nu}$  is used to adjust for locations with latitudes above 54 ° North.



Figure 3 - Seasonal Melt Factor Variation used by SNOW-17

$$M_f = \frac{\Delta t_t}{6} \cdot \{S_v \cdot A_v \cdot (MFMAX - MFMIN) + MFMIN\}$$
(8)

$$S_v = 0.5 \cdot \sin\left(\frac{N \cdot 2\pi}{366}\right) + 0.5$$
 (9)

where:  $M_f = \text{non-rain melt factor measured in } mm/^{\circ}C/\Delta t_t$ 

N = day number since March 21st,

**MFMAX** = maximum melt factor – June 21st (mm/°C/6 hrs),

**MFMIN** = minimum melt factor – Dec.  $21_{st}$  (mm/°C/6 hrs), and

A<sub>v</sub> = seasonal variation adjustment:

When latitude  $< 54^{\circ}$  North,  $A_v = 1.0$ , and

When latitude  $\geq 54^{\circ}$  North:

 $A_v = 0.0$  from September 24 to March 18,

 $A_v = 1.0$  from April 27 to August 15, and

Av varies linearly between 0.0 and 1.0 from 3/19-4/26 and between 1.0 and 0.0 from 8/16-9/23.

The total non-rain melt  $M_{nr}$  is calculated using Eq. 10. The parameter MBASE is used to vary the temperature above which melt typically occurs. The value of 0°C is used typically but is varied in special locations where physiographic conditions at the site affect the melt start temperatures.

$$M_{nr} = M_f \cdot (T_a - MBASE) \cdot \left(\frac{\Delta t_p}{\Delta t_t}\right) + 0.0125 \cdot P \cdot f_r \cdot T_r$$
(10)

where:  $M_{nr}$  = melt during non-rain periods (mm),

 $M_f = melt factor (mm/°C/\Delta t_t),$ 

 $\Delta tt$  = time interval of temperature data (hours), and

**MBASE** = base temperature ( $^{\circ}$ C).

#### 2.3.3 Energy Exchange when No Surface Melt

SNOW-17 uses a heat deficit to keep track of the net heat loss from the snow cover. The thermal gradient in the upper layer of the snowpack are estimated using the Antecedent Temperature Index, ATI, in Eq. 11. The snow surface temperature is assumed to be 0°C or air temperature, whichever is less.

$$ATI_2 = ATI_1 + TIPM_{\Delta t_t} \cdot (T_a - ATI_1) \tag{11}$$

where: ATI = antecedent temperature index (°C) where:

if  $ATI > 0^{\circ}C$ ,  $ATI = 0^{\circ}C$ ,

if  $P_n > 1.5 \cdot \Delta t_p$ ,  $ATI = T_n$ ,

 $TIPM_{\Delta t_t} = 1.0 - (1.0 - TIPM)^{\Delta t_t/6}$ , and

**TIPM** = model parameter (>0.0 and <1.0).

The equation weighs the most recent air temperatures by decreasing amounts as one goes further back in time. When there is sufficient new snowfall (greater than 1.5 mm/hr water equivalent), ATI becomes equal to the temperature of the new snow.

The heat deficit is calculated using Eq. 12. The gradient in the upper layers of the snow cover is estimated as the difference between  $T_{sur}$  and ATI. When  $T_{sur}$  is less than ATI, the heat deficit is increasing and when  $T_{sur}$  is greater than ATI the heat deficit is decreasing. The rate of the increase or decrease is based on a negative melt factor, **NMF**. The **NMF** is assumed to vary seasonally since typically the density of the snow cover tends to increase from the accumulation period to the melt season and the thermal conductivity of the snow is closely related to the density. Since the rate of heat gain or loss when the air temperature is below freezing is significantly less than when surface melt is occurring due to the insulating properties of snow and since the model uses a rough approximation to the temperature gradient in the upper layers of the snow, a unique seasonal variation is not used. Instead the seasonal variation in the negative melt factor is assumed to be the same as for the non-rain melt factor,  $M_{f}$ .

$$\Delta D_t = NM_f \cdot (ATI - T_{sur}) = NMF \cdot \left(\frac{\Delta t_p}{6}\right) \cdot \frac{M_f}{MFMAX} \cdot (ATI - T_{sur})$$
(12)

where:  $\Delta D_t$  = change in heat deficit due to a temperature gradient (mm),

 $NM_f$  = negative melt factor (mm/°C/ $\Delta t_p$ ), and

**NMF** = maximum negative melt factor (mm/ $^{\circ}C/6$  hr).

## 2.4 Internal Changes within Snow Pack

SNOW-17 only deals with the overall state of the snow cover, and does not try to calculate the temperature, liquid water or density profile within the pack. The overall ripeness is accounted for by keeping track of the heat deficit and liquid water storage.

#### 2.4.1 Snow Cover Ripeness

A snow cover is considered to be ripe when any additional melt or rain water cannot be held within the snow but will move through the pack and become outflow. This occurs when the snow cover is isothermal at 0°C and the liquid water storage capacity is full. In SNOW-17 the snow cover is ripe when both the heat deficit is zero and the amount of liquid water held in the pack equals the holding capacity. The liquid water holding capacity in SNOW-17 is determined by Eq. 13. **PLWHC** represents the overall liquid water holding capacity of a well-aged snow cover.

$$W_{ax} = PLWHC \cdot W_i \tag{13}$$

where:  $W_{qx} =$ liquid water capacity (mm),

**PLWHC** = percent liquid water holding capacity (decimal fraction), and

W<sub>i</sub> = water equivalent of the ice portion of the snow cover (mm).

The heat deficit allows the model to reasonably represent the ripening process without having to make assumptions as to whether heat losses are refreezing liquid water or lowering the temperature of the snow cover or both. SNOW-17 goes through the accounting process for the heat deficit and liquid water storage for each precipitation data time interval. The sequence is as follows:

1. The amount of liquid water available at the surface of the snow cover due to melt and rain is calculated and heat deficit is adjusted due to the temperature of the new snowfall and heat transfer caused by a temperature gradient in the upper layers of the snow cover:

$$Q_w = M_r + M_{nr} + P \cdot f_r \tag{14}$$

$$D_2 = D_1 + \Delta D_p + \Delta D_t \tag{15}$$

where:  $Q_w =$  liquid water available at the snow surface (mm), and

D = heat deficit (mm)

 $\Delta D_p$  = change in the heat deficit due to snowfall (mm) (Section 2.3.1)

 $\Delta D_t$  = change in heat deficit due to a temperature gradient (mm) (Section 2.3.3)

2. If there is sufficient water available at the surface to overcome the heat deficit and exceed the liquid water storage capacity, the snow cover becomes ripe and the excess water will be available to move through the pack and become outflow. The amount of excess water in this case is defined by Eq. 16.

$$E = Q_w + W_q - W_{qx} - D - (\mathbf{PLWHC} \cdot D)$$
(16)

where: E = excess liquid water (mm), and

 $W_q =$  liquid water held by the snow (mm).

In addition, the amount of liquid water held by the snow,  $W_q$ , is equal to the liquid water storage capacity,  $W_{qx}$ ; the amount of ice in the snow,  $W_i$ , is increased by the heat deficit, D, since that much water 'refroze' in order to raise the temperature of the pack to 0°C; and the heat deficit becomes zero. These changes are summarized by Eq. 17.  $Q_f$  is the amount of water that re-froze in this time interval.

$$W_q = W_{qx}$$

$$W_i = W_i + D$$

$$Q_f = D$$

$$D = 0$$
(17)

3. If there is only sufficient water available at the surface to overcome the heat deficit, but not enough to fill the liquid water holding capacity of the snow cover, then the new amount of liquid water is computed by Eq. 18. Q<sub>w</sub> is the amount of water available. The amount of ice in the snow, W<sub>i</sub>, is again increased by the heat deficit, due to liquid water 'refreezing' in the pack, the heat deficit becomes zero and there is no excess water available. The snowpack is not yet ripe.

$$W_q = W_q + Q_w - D$$

$$W_i = W_i + D$$

$$Q_f = D$$

$$D = 0$$
(18)

4. If there is not enough surface water to overcome the heat deficit, then the heat deficit, D, is reduced by the amount of available water, Q<sub>w</sub>; the amount of ice in the pack, W<sub>i</sub>, is increased by

the amount of water that 'refroze',  $Q_w$ ; the amount of liquid water held in the pack,  $W_q$ , remains the same; and there is no excess water available. Again the snow cover is not yet ripe.

$$W_{i} = W_{i} + Q_{w}$$

$$W_{q} = W_{q}$$

$$Q_{f} = Q_{w}$$

$$D = D - Q_{w}$$
(19)

Note: In an actual snow cover, the 'refreezing' of liquid water doesn't all occur during the same time interval as in the model. However, the net amount of surface water that refreezes within the actual snow cover during a ripening period is believed to be generally close to that computed by the model. Differences occur depending on initial snowpack conditions and how fast the ripening occurs.

#### 2.4.2 Snow Depth Calculation

The model separates new snowfall from the snow that existed at the start of the computational interval. The change in density of existing snowfall is calculated using the analytical solution in Eqs. 20 to 22. They account for compaction, destructive metamorphism, and the component of melt metamorphism resulting from the presence of liquid water. Constructive metamorphism is not included since it only changes the density profile of a snow cover and SNOW-17 treats the entire snow cover as a single entity.

$$\rho_{x2} = \rho_{x1} \cdot \left(\frac{e^{B \cdot 0.1 \cdot W_{ix}} - 1}{B \cdot 0.1 \cdot W_{ix}}\right) \cdot e^A \tag{20}$$

$$B = c_1 \cdot \Delta t_t \cdot e^{0.08 \cdot T_s - c_2 \cdot \rho_x} \tag{21}$$

$$A = c_3 \cdot c_5 \cdot \Delta t_t \cdot e^{c_4 \cdot T_s - c_x \cdot \beta \cdot (\rho_x - \rho_d)}$$
(22)

Where:  $\rho_x$  = density of the ice portion of the existing snow cover (gm·cm<sup>-3</sup>),

 $T_s$  = average snow cover temperature (°C) (assumed equal to snow surface temperature),

 $\beta = 0.0$  if  $\rho_x \le \rho_d$  and =1.0 if  $\rho_x > \rho_d$ ,

Wix = remaining ice portion of the snow cover that existed at the start of the period (mm), and

c1, c2, c3, c4, c5, cx and pd are constants defined in Anderson [1976]:

- $c1 = fractional increase in density 0.026 cm 1 \cdot hr^{-1}$
- c2 = constant estimated by Kojima [1967] 21 cm<sup>3</sup> gm<sup>-1</sup>
- c3 = fractional settling rate at 0°C for  $\rho x < \rho d 0.005$  hr<sup>-1</sup>
- $c4 = constant 0.10 \ ^{\circ}C^{-1}$

c5 = increase in fractional settling rate when liquid water exists

= 0 when Wqt = 0.0, and

= 2.0 when Wqt > 0.0.

Wqt = total liquid water in snow (mm) - (see section on Transmission of Water through the Snow Cover)

 $\rho_d$  = threshold density above which destructive metamorphism decreases – 0.20 gm·cm<sup>3</sup>

 $c_x = destructive metamorphism decay factor when \rho_x > \rho_d - 23$ .

The depth of the snow that existed at the start of the computational time interval is computed using Eq. 23.

$$H_x = \frac{W_{ix}}{\rho_x} \tag{23}$$

where:  $H_x = depth$  of the snow that existed at the start of a computational interval (mm).

Wix is the original ice content of the snow pack

 $\rho_x$  is the new density of the snow pack

Next, the snow that was present at the start of the time interval is combined with any new snowfall during the period to get the average density of the total snowpack, as shown in Eq. 24.

$$\rho_1 = \frac{W_i}{H_x + H_n} \tag{24}$$

The increase in density due to the component of melt metamorphism resulting from melt-freeze cycles is then computed based on the amount of surface melt or rain water that refroze within the snow cover using Eq. 25. (Water equivalent is increased in this case with no change in depth).

$$\rho_2 = \rho_1 \cdot \frac{W_i}{W_i - Q_f} \tag{25}$$

where:  $\rho$  = average density of the ice portion of the total snow cover (the maximum allowed value is 0.6 since the maximum value of **PLWHC** is 0.4. Thus, the lesser either  $\rho_2$  and 0.6 is selected),

 $Q_f$  = total water that refroze within the snowpack over  $\Delta t_t$  (mm).

In Eqns. 20 and 21, the temperature  $T_s$  is calculated using the following equations: 26-30.

$$T_{x,t+\Delta t_t} = T_{x,t} + \Delta T_a \cdot \frac{1.0 - e^{-\alpha \cdot 0.01 \cdot H_x}}{\alpha \cdot 0.01 \cdot H_x}$$
(26)

where:  $\Delta T_a = T_{a,t}-T_{a,t-\Delta t}$ 

if 
$$T_{a,t-\Delta t} > 0$$
 and  $T_{a,t} > 0$ ;  $\Delta T_a = abs(\Delta T_a)$ 

if  $T_{a,t-\Delta t} > 0$  and  $T_{a,t} < 0$ ;  $\Delta T_a = T_a$ 

$$\alpha = \sqrt{\frac{\pi \cdot c}{\lambda \cdot 2 \cdot 3600 \cdot \Delta t_t}}$$
(27)

where:  $\lambda$  = thermal conductivity of snow (watts·m<sup>-1</sup>·°C<sup>-1</sup>) estimated from Djachkova's formula c = effective specific volumetric heat capacity of snow (watts·sec·m<sup>-3</sup>·°C<sup>-1</sup>)

$$c = c_c \cdot \rho_x + c_a \cdot \left(1.0 - \rho_x - \theta_q\right) + c_q \cdot \theta_q$$
<sup>(28)</sup>

where:  $c_c =$  volumetric heat capacity of ice  $(2.1 \cdot 10^6)$ ,

- $c_a =$  volumetric heat capacity of air (1.0·10<sup>3</sup>),
- $c_q$  = volumetric heat capacity of water (4.2·10<sup>6</sup>), and
- $\theta_q$  = fraction of liquid water in snow Wqt/(Wi+Wqt)

 $T_x$  = average temperature of the existing snow cover (°C).

When there is new snowfall during the computational period, Eq. 26 is modified to Eq. 29 to take into account the insulating effect of the new snow.

$$T_{x,t+\Delta t} = T_{x,t} + \Delta T_a \cdot \left[ \frac{e^{-\alpha \cdot 0.01 \cdot H_n} - e^{-\alpha \cdot 0.01 \cdot H_x}}{\alpha \cdot 0.01 \cdot (H_x - H_n)} \right]$$
(29)

The weighted average snow cover temperature is then computed using Eq. 30.

$$T_s = \frac{(T_x \cdot H_x) + (T_n \cdot H_n)}{H_x + H_n}$$
(30)

Originally, the SnowDepth software implemented the snow depth densification by interpolating data points experimentally found between Snow Depth and SWE. Figure 4 shows the results. Both methods show similar level of accuracy but the equations defined in this section were chosen ultimately for the SnowDepth software as they follow the trend more accurately.



Figure 4 - Comparison between interpolation method and equations

# 3 SnowDepth Calibration

Bounded constrained multivariable optimization, using fminsearchcon, is applied to calibrate the model [3]. It adapts the MATLAB fminsearch function which is a nonlinear programming solver to allow for bounds [4].

## 3.1 Summary of Parameters

This section provides expected values for each input parameter [5] [6].

Parameters	s Expected Value		Comments
PXTEMP	<b>XTEMP</b> Recommended 0.5-21 Degree Adequate3-5 for Mountainous Regions		Currently set from 0.5-2 Currently this parameter decides whether it's 100% snow or 100% rain.
SCF	<ul> <li>1.1-1.2 Reasonable</li> <li>1.3-1.6 For locations with poor gauge exposure</li> <li>1.0 or a bit less for good exposure and significant sublimation losses</li> </ul>		Currently set to 0.95-1.6 Did not vary significantly when optimizing for different years on the same location.
MFMIN	Dense conifer forest or persistent cloud cover Mixed cover - conifer, deciduous, open Mostly deciduous Mostly Open: flat terrain Mostly Open: mountainous terrain	0.2-0.4 0.1-0.3 0.2-0.6 0.2-0.6 0.1-0.3	Currently set between 0.1-0.6 Use Table to Left to check if values are reasonable
MFMAX	Dense conifer forest or persistent cloud cover Mixed cover - conifer, deciduous, open Mostly deciduous Mostly Open: flat terrain Mostly Open: mountainous terrain	0.5-0.7 0.8-1.2 1.0-1.4 1.5-2.2 0.9-1.3	Currently set 0.5-1.5 Use Table to Left to check if values are reasonable

3.1.1 Major Snow Parameters:

	Usually between 0.05 (2.5 mi/hr wind speed) to 0.20 (10 mi/hr wind speed).	
UADJ	Can be estimated using:	Currently set to 0.05-0.2 Output is not very sensitive
CILIS	$UADJ = 0.002 \cdot u1$	Sulput is not very sensitive
	(7-4-1)	
	where: $u1 = 6$ hr. wind travel in km at a 1 meter height above the snow surface.	

## 3.1.2 Minor Snow Parameters

Parameters	Expected Value	Comments
	Recommended:	
	0.05 for regions with deep snow cover $(> 3 \text{ ft for most years})$	
TIPM	0.20 for regions with shallow snow cover (< 1ft )	Currently set from 0.05-0.20
	Intermediate values in between	
	Reasonable value: 0.15 mm/°C/6 hr	
	(based on maximum snow density of 0.	
	3 for shallow snow cover and 0.5 for a deep pack.)	
NMF	If maximum density is less than suggested values, decrease NMF	Currently set from 0.05-0.3
	If maximum density is greater than suggested values, increase NMF	
	Reasonable range: 0.05-0.3	
	Regions with a variety of vegetation cover, slopes, and aspects, the value of MBASE is almost always 0°C.	
	If non-zero value is required for a good fit, it indicates bias errors in average temperature date	Should set to 0 or let user
MBASE	Case for larger than 0 MBASE:	choose.
WIDASE	1. high elevation, open areas with generally clear skies and relatively low humidity during the melt season.	Currently set
	2. open, high elevation point location such as a snow course site	from 0-1
	3. when modeling other point locations due to site specific factors that control the relationship between measured air temperature and melt amounts	

	According to studies, should be within 0.02-0.05		
PLWHC	Lowest values for deep snow covers	Set between	
	To account for the slush layer in areas with shallow snow covers, especially in the plains and open agricultural areas of the Midwest, a range of 0.1 to 0.3 is common.	0.02-0.05	
	DAYGM = 0.0 for areas with generally frozen soils under the snow, and		
DAYGM	DAYGM = 0.3 for areas with intermittent snow cover or with fairly temperate climates, such as the Sierra Nevada mountains in California, during the winter.	Not Required for SnowDepth	
	Other areas will have values in between		

# 4 Validation

SnowDepth was validated for multiple locations over multiple years of data sourced from the United States Department of Agriculture [7] [8]. In this section, the results for a multiple year comparison for the Lone Mountain location is provided.

The model parameters were initially optimised for the 2010-2011 data. Two separate least-square optimizations were conducted: one to minimize the error between modelled and measured SWE; and the other to minimize error between measured and modelled Snow Depth.

2010-2011 Data Plots Optimized for SWE



2010-2011 Data Plots Optimized for Snow Depth







	Parameters optimized for SWE	Parameters optimized for Snow Depth
PXTEMP	<mark>1.6615</mark>	<mark>0.5739</mark>
SCF	1.3082	1.3941
UADJ	0.1891	0.1908
NMF	<mark>0.0843</mark>	0.2288
MFMIN	0.2794	0.1780
MFMAX	1.3033	1.4694
MBASE	<mark>0.9902</mark>	<mark>0.0004</mark>
TIPM	0.0959	0.1041
PLWHC	0.0491	0.0398

The major differences in parameters are highlighted. The optimized parameters calculated using the 2010-2011 data were used to predict snow melt for the 2011-2015 water cycle years for Lone Mountain. The results are below:



## 2011-2012 Data Plots for 2010-2011 SWE Optimized Parameters

2011-2012 Data Plots for 2010-2011 Snow Depth Optimized Parameters



## 2012-2013 Data Plots for 2010-2011 SWE Optimized Parameters



2012-2013 Data Plots for 2010-2011 Snow Depth Optimized Parameters



## 2013-2014 Data Plots for 2010-2011 SWE Optimized Parameters



2013-2014 Data Plots for 2010-2011 Snow Depth Optimized Parameters





## 2014-2015 Data Plots for 2010-2011 SWE Optimized Parameters



2014-2015 Data Plots for 2010-2011 Snow Depth Optimized Parameters





## 4.1 Discussion

The model predictions worsen when you use the optimized parameters with later years. That is, the SWE prediction for 2014-2015 was the worst. However, the predictions for 2011-2012 and 2012-2013 were relatively accurate. In addition, the predictions tend to be slightly more accurate when optimizing for SWE as compared to Snow Depth. If optimized for the year 2013-2014, the following results are retrieved.

3000

2000

1000

Oct Nov Dec Jan Feb Mar

Snow depth [mm]

### 2013-2014 Data Plots Optimized for SWE



2013-2014 Data Plots Optimized for Snow Depth



May Jun Jul Aug

Ap

Date

Modeled

Observed Daily Precipitatio

Precipitation [mm]

10





	Parameters optimized for SWE For 2013-2014	Parameters optimized for SWE For 2010-2011
PXTEMP	1.6171	1.6615
SCF	1.2354	1.3082
UADJ	<mark>0.0885</mark>	<mark>0.1891</mark>
NMF	0.1135	0.0843
MFMIN	<mark>0.1000</mark>	<mark>0.2794</mark>
MFMAX	<mark>0.8877</mark>	<mark>1.3033</mark>
MBASE	0.9999	0.9902
TIPM	0.1225	0.0959
PLWHC	0.0500	0.0491

### Comparison of SWE Optimized Parameters for 2010-2011 vs. 2013-2014 Data

Major Parameter changes highlighted.

MFMIN and MFMAX are within reason for a Mixed-cover conifer, deciduous, open location; though are probably the factors that mainly affecting the model in this case.

	Parameters optimized for Snow Depth for 2013-2014	Parameters optimized for Snow Depth for 2010-2011
PXTEMP	<mark>1.1103</mark>	<mark>0.5739</mark>
SCF	1.4179	1.3941
UADJ	0.1438	<mark>0.1908</mark>
NMF	0.2095	0.2288
MFMIN	<mark>0.1037</mark>	<mark>0.1780</mark>
MFMAX	<mark>1.1543</mark>	<mark>1.4694</mark>
MBASE	0.1542	0.0004
TIPM	0.1152	0.1041
PLWHC	0.0299	0.0398

## Comparison of Snow DepthOptimized Parameters for 2010-2011 vs. 2013-2014 Data

The major changes are similar to the SWE optimization apart from PXTEMP. This might be because the snow compaction routine is highly dependent on the ice content of the snow Wi which in turn depends on PXTEMP, the parameter which determines if snow or rain has fallen. Predictions might be improved if it is set to a constant and the user can alter whether rain or snow has fallen for specific events (not implemented in SnowDepth).

If the new parameters are used to calculate 2013-2014 data, the predictions are significantly improved as shown below. The optimization using SWE usually gives relatively accurate SWE and Snow Depth predictions. With Snow Depth optimizations the prediction for SWE suffers which worsens the snow depth predictions.

Overall, it is concluded that minimizing the error for SWE is more robust in comparison to Snow Depth; as a result, SWE is used for the optimization in the SnowDepth software. In addition, accurate predictions are only possible for subsequent years; that is, if a prediction is required for the 2015-2016 hydrological year, data from the 2014-2015 should be used to calibrate the model parameters ideally. Prior years can be used but might be less accurate.



#### 2014-2015 Data Plots for 2013-2014 SWE Optimized Parameters

## **5** References

- [1] E. Anderson, "Snow Accumulation and Ablation Model SNOW-17," January 2016. [Online]. Available: http://www.nws.noaa.gov/oh/hrl/nwsrfs/users\_manual/part2/\_pdf/22SNOW-17.pdf.
- [2] N. R. Hedstrom and J. W. Pomeroy, "Measurements and modelling of snow interception in the boreal forest," *Hydrology Proceedings 12*, pp. 1611-1625, 1998.
- [3] J. D'Errico, "fminsearchcon," 6 February 2012. [Online]. Available: https://www.mathworks.com/matlabcentral/fileexchange/8277-fminsearchbnd-fminsearchcon/content/FMINSEARCHBND/fminsearchcon.m.
- [4] MathWorks, "fminsearch," [Online]. Available: https://www.mathworks.com/help/matlab/ref/fminsearch.html.
- [5] "Initial Parameter Values for the SNOW-17 Model," [Online]. Available: http://www.nws.noaa.gov/oh/hrl/calb/calibration1102/calb\_report\_section7-4.pdf.
- [6] E. Houle and J. Kaspryzyk, "Investigating Parameter Sensitivity for Management in Snow-Driven Watersheds," 2014. [Online]. Available: http://www.iemss.org/sites/iemss2014/papers/iemss2014\_submission\_47.pdf.
- [7] United States Department of Agriculture, "Report Generator," [Online]. Available: http://wcc.sc.egov.usda.gov/nwcc/yearcount?network=sntl&state=&counttype=statelist. [Accessed 1 May 2016].
- [8] United States Department of Agriculture, "NRCS National Water Climate Center Active SNOTEL Stations as of 2016-September-29," [Online]. Available: http://wcc.sc.egov.usda.gov/nwcc/yearcount?network=sntl&state=&counttype=statelist. [Accessed 1 May 2016].